3. Derivacije i primjene - 1. dio

1. Derivirajte funkcije:

    (a) $f(x) = x^2 + x^3 + \sin x$;

    (b) $f(x) = (x^2 - x + 1)(x^4 + 2)$;

    (c) $f(x) = \dfrac{x}{x^2 + 1}$;

    (d) $f(x) = \sqrt{x} + \pi$;

    (e) $f(x) = \sqrt[3]{x^2} - \sqrt{x\sqrt{x}} + \sqrt[7]{e}$;

    (f) $f(x) = \dfrac{1 - \sqrt{x}}{1 + \sqrt[3]{x}}$.

2. Derivirajte funkcije:

    (a) $f(x) = \sin x + \operatorname{tg} x$;

    (b) $g(x) = \cos x \cdot (1 + \operatorname{ctg} x)$;

    (c) $h(x) = \dfrac{\cos x}{1 + \operatorname{tg} x}$;

    (d) $u(x) = \arcsin x + \arccos x$;

    (e) $v(x) = \sin x \cdot \operatorname{arctg} x$.

3. Derivirajte funkcije:

    (a) $f(x) = e^x + 2^x + \left(\dfrac{2}{3}\right)^x$;

    (b) $g(x) = \ln x + \log x$;

    (c) $h(x) = \dfrac{\sin x}{x^3} + e^x \cdot \cos x - (x^3 + 2)\log x$;

    (d) $u(x) = \dfrac{\operatorname{ctg} x}{x \ln x} + 3xe^x$.

4. Derivirajte kompozicije funkcija

    (a) $f(x) = \left(x^2 + 2\right)^3$;

    (b) $f(x) = \ln(\sin x)$;

    (c) $f(x) = \sqrt{xe^x}$;

(d) $f(x) = \sqrt{4x - 1} + \text{arcctg}\,\sqrt{4x - 1}$;

(e) $f(x) = \sqrt{x^2 - e^x} + \arcsin\dfrac{1}{x}$;

(f) $f(t) = \dfrac{1}{4}(\text{tg}\,t)^4 - \dfrac{1}{2}(\text{tg}\,t)^2 - \ln(\cos t)$;

(g) $f(x) = \arcsin\dfrac{x - 1}{x}$;

(h) $f(x) = [\ln(2x + 1)]^2$;

(i) $f(x) = \ln\sqrt{\dfrac{1 + x}{1 - x}}$;

(j) $f(x) = \dfrac{x^2 + \sin 2x}{\ln x + \cos(2x + 3)}$;

(k) $f(x) = e^{-x} + 2^{sin\frac{x}{2}} + \sin^2 x$;

(l) $f(x) = 2^{\text{arctg}\,\sqrt{x}}$;

(m) $f(x) = \ln\left(x - \sqrt{x^2 + 1}\right)$;

(n) $f(x) = x \cdot \sqrt{\text{tg}\,x}$;

(o) $f(t) = \left(\dfrac{t - 2}{2t + 1}\right)^9$;

(p) $f(x) = \text{arctg}(\ln x) + \sqrt{\ln(x^2 + 1)}$;

(q) $f(x) = \dfrac{1}{5^{x^2}}$;

(r) $f(x) = e^{\sqrt{xe^x}}$.

5. Derivirajte funkcije:

(a) $f(x) = x^{\sin x}$;

(b) $f(x) = \dfrac{(x^2 + 2x + 3)^{15}(2x + 5)^{10}}{(5x - 9)^{13}}$;

(c) $f(x) = (\ln x)^x$;

(d) $f(x) = \dfrac{(\cos x)^{\sin x}}{x^2 + 3}$;

(e) $f(x) = \ln\sqrt[x]{\sin x}$;

(f) $f(x) = e^{\cos x} + (\cos x)^x$;

(g) $f(x) = \dfrac{\sqrt{(x-2)(x-4)}}{(x+1)(x+3)}$;

(h) $f(x) = \sqrt[x]{x}$.

6. Derivirajte funkcije:

(a) $x^3 y + xy^3 = e^x$;

(b) $xy + \sin y = e^{x+y}$;

(c) $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$;

(d) $\sqrt{x} + \sqrt{y} = \sqrt{2e}$;

(e) $\left(x^2 + y^2\right) \cdot y^2 = a \cdot x^2$.

7. Izračunajte n-tu derivaciju funkcije i njenu vrijednost u x_0:

(a) $f(x) = x^5$, $x_0 = 0$;

(b) $f(x) = \dfrac{1}{x}$, $x_0 = -1$;

(c) $f(x) = \cos x$, $x_0 = \pi$;

(d) $f(x) = \sin x$, $x_0 = \frac{\pi}{2}$;

(e) $f(x) = \ln \dfrac{1 - 4x}{1 + 4x}$, $x_0 = 0$.

8. Derivirajte parametarski zadane funkcije:

(a) $x(t) = t - \sin t$,
    $y(t) = 1 - \cos t$;

(b) $x(t) = \frac{t(t+1)}{t+2}$,
    $y(t) = \frac{t^2 - 4t + 1}{t}$;

(c) $x(t) = \sqrt{\sin 3t}$,
    $y(t) = \left(\frac{t+3}{t-3}\right)^5$.