## 5. Funkcije više varijabli, 2. dio - Rješenja

1. (a) $dz = (2x + y)\,dx + (x - 2y)\,dy$

   (b) $dz = \dfrac{1}{x+y}dx - \dfrac{x}{y\,(x+y)}dy$

2. (a) $d^2z = -\frac{1}{(x+y)^2}\,(dx + dy)^2$

   (b) $d^2z = -\frac{1}{x}dx^2 + \frac{2}{y}dxdy - \frac{x}{y^2}dy^2$

3. (a) $\dfrac{dz}{dt} = \dfrac{e^t\,(t \ln t - 1)}{t \ln^2 t}$

   (b) $\dfrac{dz}{dt} = e^{\sin t - 2t^3}\left(\cos t - 6t^2\right)$

   (c) $\dfrac{dz}{dt} = \dfrac{3 - 12t^2}{\sqrt{1 - (3t - 4t^3)^2}}$

   (d) $\dfrac{dz}{dt} = \dfrac{3 - \frac{4}{t^3} - \frac{1}{2\sqrt{t}}}{\cos^2\left(3t + \frac{2}{t^2} - \sqrt{t}\right)}$

4. (a) $\dfrac{\partial z}{\partial u} = \dfrac{2u}{v^2}\ln(3u - 2v) + \dfrac{3u^2}{v^2\,(3u - 2v)}$

   $\dfrac{\partial z}{\partial v} = -\dfrac{2u^2}{v^3}\ln(3u - 2v) - \dfrac{2u^2}{v^2\,(3u - 2v)}$

   (b) $\dfrac{\partial z}{\partial u} = 2ue^{uv}\left(u^2 - v^2\right)^{e^{uv}-1} + ve^{uv}\left(u^2 - v^2\right)^{e^{uv}} \ln\left(u^2 - v^2\right)$

   $\dfrac{\partial z}{\partial v} = -2ve^{uv}\left(u^2 - v^2\right)^{e^{uv}-1} + ue^{uv}\left(u^2 - v^2\right)^{e^{uv}} \ln\left(u^2 - v^2\right)$

   (c) $\dfrac{\partial z}{\partial u} = 2u\cos u^2 - \dfrac{2uv^2\left(1 + v^4\right)}{\left(v^2 + u^2v^4 + u^2\right)^2}, \dfrac{\partial z}{\partial v} = \dfrac{2u^2v\left(1 - v^4\right)}{\left(v^2 + u^2v^4 + u^2\right)^2}$

   (d) $\dfrac{\partial z}{\partial u} = 0, \dfrac{\partial z}{\partial v} = 1$

5. (a) $y' = \dfrac{y}{x}, y'' = 0$

   (b) $y' = -\dfrac{x}{y}, y'' = -\dfrac{x^2 + y^2}{y^3}$

6. (a) $\dfrac{\partial z}{\partial x} = -\dfrac{c^2x}{a^2z}, \dfrac{\partial z}{\partial y} = -\dfrac{c^2y}{b^2z}$

(b) $\dfrac{\partial z}{\partial x} = \dfrac{z}{x\,(z-1)}, \dfrac{\partial z}{\partial y} = \dfrac{z}{y\,(z-1)}$

7. (a) $dz = -\dfrac{1}{z}(dx + dy)$

   (b) $dz = \dfrac{(yz-1)\,dx + (xz-1)\,dy}{1-xy}$

8. $R_t \ldots 2x{-}4y{-}z{-}5 = 0, n \ldots \dfrac{x-1}{2} = \dfrac{y+2}{-4} = \dfrac{z-5}{-1}$

9. $R_t \ldots 2x{-}2y{+}4z{-}\pi = 0, n \ldots \dfrac{x-1}{1} = \dfrac{y-1}{-1} = \dfrac{z-\frac{\pi}{4}}{2}$

10. U tim točkama imaju istu tangencijalnu ravninu.