

2. Tehnike integriranja, 2. dio - Rješenja

$$1. I_n = -\frac{1}{n} \cos x \sin^{n-1} x + \frac{n-1}{n} I_{n-2}, \quad n \geq 2,$$
$$I_4 = -\frac{1}{4} \sin^3 x \cos x - \frac{3}{8} \sin x \cos x + \frac{3}{8} x + c.$$

$$2. I_n = \frac{x(a^2 - x^2)^n}{2n+1} + \frac{2na^2}{2n+1} I_{n-1}.$$

$$3. I_n = x \ln^n x - n I_{n-1}.$$

$$4. I_n = \frac{1}{a} x^n e^{ax} - \frac{n}{a} I_{n-1}.$$

$$5. I = 6 \ln \sqrt[6]{x} - \frac{3}{2} \ln (\sqrt[6]{x} + 1) - \frac{9}{4} \ln (2\sqrt[3]{x} - \sqrt[6]{x} + 1) -$$
$$\frac{3}{2\sqrt{7}} \operatorname{arctg} \frac{4\sqrt[6]{x} - 1}{\sqrt{7}} + c.$$

$$6. I = \frac{3}{2} \sqrt[3]{x^2} + \operatorname{arctg} \sqrt[6]{x} + c.$$

$$7. I = 2\sqrt{x} - 2 \ln |1 + \sqrt{x}| + c.$$

$$8. I = \frac{3}{2} (2x+1)^{\frac{1}{3}} + 3(2x+1)^{\frac{1}{6}} + 3 \ln |\sqrt[6]{2x+1} - 1| + c.$$

$$9. I = -\frac{6}{7} (x+1)^{\frac{7}{6}} + \frac{6}{5} (x+1)^{\frac{5}{6}} + \frac{3}{2} (x+1)^{\frac{2}{3}} - 2(x+1)^{\frac{1}{2}} - 3(x+1)^{\frac{1}{3}} + 6(x+1)^{\frac{1}{6}} + 3 \ln |\sqrt[3]{x+1} + 1| -$$
$$6 \operatorname{arctg} \sqrt[6]{x+1} + c.$$

$$10. I = \ln |\sqrt{x^2 + 2x + 2} + x + 1| + \frac{2}{\sqrt{x^2 + 2x + 2} + x + 2} + c.$$

$$11. I = \frac{10}{9} \cdot \frac{x-2}{\sqrt{7x-10-x^2}} - \frac{4}{9} \cdot \frac{\sqrt{7x-10-x^2}}{x-2} + c.$$

$$12. I = 2 \ln |\sqrt{x^2 - x + 1} - x| - \frac{3}{2} \ln |2\sqrt{x^2 - x + 1} -$$
$$2x + 1| + \frac{3}{2} \cdot \frac{1}{2\sqrt{x^2 - x + 1} - 2x + 1} + c.$$

$$13. I = -\frac{2}{1 + \sqrt{\frac{1-x}{1+x}}} - 2 \operatorname{arctg} \sqrt{\frac{1-x}{1+x}} + c$$

$$14. I = \ln \left| \frac{\sqrt{1-2x-x^2}-x+1}{\sqrt{1-2x-x^2}+1} \right| - 2 \operatorname{arctg} \frac{\sqrt{1-2x-x^2}+1}{x} + c.$$

$$15. I = \left(\frac{1}{2}x - \frac{1}{4} \right) \sqrt{4x^2 - 4x + 3} + \frac{1}{2} \ln |2x-1+\sqrt{4x^2 - 4x + 3}| + c.$$

$$16. I = 2 \arcsin \frac{x+1}{2} + \frac{x+1}{2} \sqrt{3-2x-x^2} + c.$$

$$17. I = \left(-\frac{1}{3}x^2 - \frac{5}{6}x - \frac{19}{6} \right) \sqrt{1+2x-x^2} + 4 \arcsin \frac{x-1}{\sqrt{2}} + c.$$

$$18. I = \left(-\frac{1}{2}x - 5 \right) \sqrt{-x^2 + 4x} + 13 \arcsin \frac{x-2}{2} + c.$$

$$19. I = \left(\frac{1}{3}x^2 + \frac{1}{6}x + \frac{7}{6} \right) \sqrt{x^2 + 2x + 2} + \frac{5}{2} \ln |x+1 + \sqrt{x^2 + 2x + 2}| + c.$$

$$20. I = -\frac{4}{3} \sqrt{1-x\sqrt{x}} + c.$$

$$21. I = -\frac{1}{2(\sqrt[4]{x}+1)^8} + \frac{4}{9(\sqrt[4]{x}+1)^9} + c.$$

$$22. I = 2(1 + \sqrt[3]{x})^{\frac{3}{2}} + c.$$

$$23. I = \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+3}}{(4n+3)(2n+1)!}.$$

$$24. \ln(1+x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1}, \quad x \in \langle -1, 1 \rangle.$$

$$25. I = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{(n+1)^2}, \quad x \in \langle -1, 1 \rangle.$$