

5. Funkcije više varijabli, 1. dio - Rješenja

1. (a) $\mathbb{R}^2 \setminus \{(0, 0)\}$
 (b) $\{(x, y) : x = y\}$
 (c) $\mathbb{R}^2 \setminus \{(x, y) : y = x^2\}$
 (d) $\{(x, y) : y > -x^2\}$
 (e) $\mathbb{R}^2$
 (f) $\{(x, y) : -2 \leq x \leq 0, y \leq 0\} \cup \{(x, y) : 0 \leq x \leq 2, y \geq 0\}$
 (g) $\{(x, y) : y \leq 0, y \leq -2x\} \cup \{(x, y) : y \geq 0, y \geq -2x\}$
 (h) $\{(x, y) : -1 \leq x \leq 1, -1 \leq y \leq 1\}$
 (i) $\{(x, y) : x^2 + y^2 \leq 1\} \cup \{(x, y) : x^2 + y^2 \geq 4\}$
 (j) $\left(\bigcup_{k \in \mathbb{Z}} \{(x, y) : y \geq 0, 2k\pi \leq x \leq (2k+1)\pi\} \right) \cup$
 $\left(\bigcup_{l \in \mathbb{Z}} \{(x, y) : y \leq 0, (2l+1)\pi \leq x \leq 2(l+1)\pi\} \right)$
 (k) $\{(x, y) : y^2 \geq 4x, x^2 + y^2 > 1, x^2 + y^2 \neq 1\}$
 (l) $\{(x, y) : x > 0, y > x + 1\} \cup \{(x, y) : x < 0, x < y < x + 1\}$
2. (a) $L = 2$
 (b) $L = a$
 (c) Limes ne postoji.
 (d) Limes ne postoji.
3. (a) $\frac{\partial z}{\partial x} = \frac{1}{\sqrt{y}} \operatorname{ctg} \frac{x+a}{\sqrt{y}}, \frac{\partial z}{\partial y} = -\frac{x+a}{2\sqrt{y^3}} \operatorname{ctg} \frac{x+a}{\sqrt{y}}$
 (b) $\frac{\partial f}{\partial x} = \frac{|y|}{y\sqrt{y^2 - x^2}}, \frac{\partial f}{\partial y} = \frac{-x|y|}{y^2\sqrt{y^2 - x^2}}$
 (c) $\frac{\partial u}{\partial x} = z(xy)^{z-1}y, \frac{\partial u}{\partial y} = z(xy)^{z-1}x, \frac{\partial u}{\partial z} = (xy)^z \ln(xy)$
 (d) $\frac{\partial f}{\partial x} = y^z x^{(y^z-1)}, \frac{\partial f}{\partial y} = (x^{y^z} \ln x)zy^{z-1}, \frac{\partial f}{\partial z} = (x^{y^z} \ln x)(y^z \ln y)$

$$4. (a) \frac{\partial^2 f}{\partial x^2} = \frac{2y - 2x^2}{(x^2 + y)^2}, \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = \frac{-2x}{(x^2 + y)^2},$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{-1}{(x^2 + y)^2}$$

$$(b) \frac{\partial^2 f}{\partial x^2} = y(y - 1)x^{y-2}, \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = x^{y-1}(1 + y \ln x),$$

$$\frac{\partial^2 f}{\partial y^2} = x^y \ln^2 x$$

$$(c) \frac{\partial^2 f}{\partial x^2} = \frac{-3xy^2}{(x^2 + y^2)^{5/2}}, \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = \frac{2x^2y - y^3}{(x^2 + y^2)^{5/2}},$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{2xy^2 - x^3}{(x^2 + y^2)^{5/2}}$$

$$5. \frac{\partial^2 f}{\partial x^2}(0, 0) = m(m - 1),$$

$$\frac{\partial^2 f}{\partial y \partial x}(0, 0) = \frac{\partial^2 f}{\partial x \partial y}(0, 0) = m \cdot n,$$

$$\frac{\partial^2 f}{\partial y^2}(0, 0) = n(n - 1)$$

$$6. (a) \frac{\partial^3 u}{\partial x \partial y \partial z} = (x^2 y^2 z^2 + 3xyz + 1)e^{xyz}$$

$$(b) \frac{\partial^3 u}{\partial x^3} = e^{xyz} y^3 z^3, \frac{\partial^3 u}{\partial y^3} = e^{xyz} x^3 z^3, \frac{\partial^3 u}{\partial z^3} = e^{xyz} x^3 y^3$$

$$7. \frac{\partial^2 z}{\partial y \partial x} = \frac{-y^2}{(2xy + y^2)^{3/2}}$$

$$8. \frac{\partial^3 z}{\partial^2 x \partial y} = 0$$

$$9. z(x, y) = 2xy + \frac{y^2}{2} + \varphi(x)$$

$$10. z(x, y) = y^2 + xy + 1$$