1 Supstitucije za $\int R\left(x, \sqrt{ax^2+bx+c}\right)dx$

Eulerove supstitucije

a) $a>0 \Longrightarrow \sqrt{ax^2+bx+c} = t \pm x\sqrt{a}$,

b) $c>0 \Longrightarrow \sqrt{ax^2+bx+c} = tx \pm \sqrt{c}$,

c) α korijen izraza $ax^2+bx+c \Longrightarrow \sqrt{ax^2+bx+c} = (x-\alpha)t$.

Trigonometrijske supstitucije

a) $\int R(x, \sqrt{a^2-x^2})dx \Longrightarrow x = a\sin t$

b) $\int R(x, \sqrt{a^2+x^2})dx \Longrightarrow x = a\,\mathrm{tg}\, t$

c) $\int R(x, \sqrt{x^2-a^2})dx \Longrightarrow x = \dfrac{a}{\cos t}$

2 Neke rekurzivne formule

1. $I_n = \displaystyle\int \frac{dx}{(x^2+a^2)^n}$, $n \geq 1 \Longrightarrow$

   $I_{n+1} = \dfrac{1}{2a^2n}\dfrac{x}{(x^2+a^2)^n} + \dfrac{1}{a^2}\dfrac{2n-1}{2n}I_n$,

   $I_1 = \dfrac{1}{a}\,\mathrm{arctg}\,\dfrac{x}{a} + C$

2. $I_n = \displaystyle\int \sin^n x\,dx$, $n \geq 0 \Longrightarrow$

   $I_n = -\dfrac{1}{n}\sin^{n-1}x\cos x + \dfrac{n-1}{n}I_{n-2}$,

   $I_0 = x + C$, $I_1 = -\cos x + C$.

3 Binomni integral

$$\int x^m(a+bx^n)^p, \quad m,n,p \in \mathbb{Q},$$

svodi se na integral racionalne funkcije u tri slučaja:

a) $p \in \mathbb{Z} \Longrightarrow x = t^k$, k je zajed. nazivnik od m i n;

b) $\dfrac{m+1}{n} \in \mathbb{Z} \Longrightarrow a+bx^n = t^k$, k je nazivnik od p;

c) $\dfrac{m+1}{n} + p \in \mathbb{Z} \Longrightarrow ax^{-n}+b = t^k$, k je naz. od p.

4 Supstitucije za $\int R(\sin x, \cos x)dx$

Univerzalna supstitucija

$$\mathrm{tg}\,\frac{x}{2} = t, \quad dx = \frac{2dt}{1+t^2},$$
$$\sin x = \frac{2t}{1+t^2}, \quad \cos x = \frac{1-t^2}{1+t^2}.$$

Pojednostavljenja

a) $R(-\sin x, \cos x) = -R(\sin x, \cos x) \Longrightarrow \cos x = t$,

b) $R(\sin x, -\cos x) = -R(\sin x, \cos x) \Longrightarrow \sin x = t$,

c) $R(-\sin x, -\cos x) = R(\sin x, \cos x) \Longrightarrow$

$$\mathrm{tg}\, x = t, \quad dx = \frac{dt}{1+t^2},$$
$$\sin x = \frac{t}{\sqrt{1+t^2}}, \quad \cos x = \frac{1}{\sqrt{1+t^2}}.$$